Statistical Thinking for Forensic Practitioners

Practice Final Exam

1. If two images of outsoles have RACs with positional similarity (i.e., located in the same position), then there is evidence that the shoes match. A study determined that the probability of selecting two outsole images at random and finding a pair of RACs with positional similarity anywhere on the outsole is 0.58. Suppose an automated RAC detection algorithm is able to correctly identify when two outsole images have RACs with positional similarity 96% of the time. It can also correctly determine when two outsole images do not share any RACs with positional symmetry 98% of the time. What is the probability that a pair of outsole images actually have RACs with positional similarity if the algorithm has determined that they do?

2. A researcher wants to conduct a study to analyze elemental concentration in a population of 1000 panes of glass.

   (a) The researcher is considering choosing a sample of 50 panes by visibly inspecting each pane and selecting those with the fewest visible markings. Which of the following best describes such a sampling scheme?

   A. Stratified Sampling

   B. Purposive Sampling

   C. Network Sampling

   D. Cluster Sampling

   (b) The researcher decides instead to use a random sampling approach to avoid visibly inspecting each pane. However, the researcher learns that the panes of glass can only be delivered in crates, each containing 25 panes. It would be logistically too challenging for them to randomly sample 50 panes across all crates. What sampling technique would you recommend the researcher change to?

   A. Cluster Sampling

   B. Stratified Sampling

   C. Network Sampling

   D. Simple Random Sampling

   (c) If the researcher decides to use Simple Random Sampling with replacement, which of the following expressions represents the total number of possible samples that could be selected?

   A. $50 * 1000$

   B. 1000^{50}

   C. 50^{1000}

   D. $\binom{1000}{50}$

   (d) The researcher considers 3 different types of glass: architectural float glass, automotive glass, and drinkware glass. Which of following sampling schemes is an example of Stratified Sampling?

   A. The researcher randomly selects one of the types of glass and measures the elemental concentrations of all samples of this type.

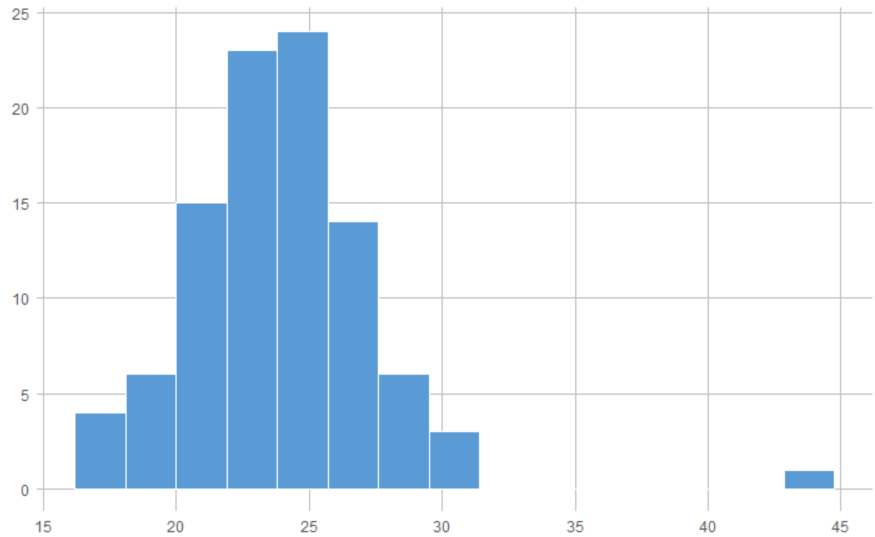
   B. The researcher randomly selects 20 samples from each glass type.

   C. The researcher selects 60 glass samples irrespective of type.

   D. The researcher selects 60 building float glass samples because they are the cheapest to obtain.

3. A forensic toxicology team is interested in analyzing the relationship between an individual's chronic alcohol drinking status (i.e., whether they are a chronic alcohol drinker) and their biomarker data (BMI, measurements from blood, hair, etc.). They collected information from 125 individuals, 29 of whom were classified as "chronic alcohol drinkers." The remaining 96 individuals were classified as "not chronic alcohol drinkers."

   (a) Consider the following histogram of BMI values for the 96 individuals classified as "not chronic alcohol drinkers."



   Which of the following best describes the shape of the distribution?

   A. Approximately uniform with no outliers.

   B. Right-skewed with no outliers.

   C. Left-skewed with outliers.

   D. Approximately symmetric with outliers.

   (b) Which of following statistic combinations would be most appropriate to describe the center and spread of the above data?

   A. Mean and Inter-Quartile Range

   B. Mean and Standard Deviation

   C. Median and Inter-Quartile Range

   D. Median and Standard Deviation

(c) The researchers are interested in identifying a probability distribution that describes the bulk of the data shown above (i.e., ignoring outlier values, if they exist). Given that BMI is a continuous variable, which of the following models is most appropriate?

A. Normal

B. Binomial

C. Log-Normal

D. Poisson

(d) Under what conditions do we use a t-distribution instead of a standard normal distribution to perform inference using the sample mean?

A. When the population mean is known.

B. When the population mean is unknown.

C. When the population variance is known.

D. When the population variance is unknown.

4. Assume that the sample size is sufficiently large to apply Normal/t-distribution based inferential procedures to this problem. The BMI cut-off for "overweight" is 25. Seventeen (17) out of the 60 subjects who have BMI greater than or equal to 25 are chronic alcohol drinkers. Construct a 99% confidence interval for the proportion of individuals with BMI greater than or equal to 25 who are chronic alcohol drinkers.

(a) Determine the appropriate critical value.

(b) Calculate the appropriate standard error.

(c) Calculate the lower and upper bounds of the 99% confidence interval.

(d) Which of the following is the most appropriate interpretation of the confidence interval calculated above?

A. The probability that the true proportion of individuals with BMI greater than or equal to 25 who are chronic alcohol drinkers is within the interval calculated is above 0.99.

B. If we were to repeat this study 100 times and calculate a confidence interval for each repetition using the same procedure as above, we would expect that 99 of these confidence intervals would contain the true proportion of individuals with BMI greater than or equal to 25 who are chronic alcohol drinkers.

C. There is a 99% chance that the true proportion of chronic alcohol drinkers who have BMI greater than or equal to 25 is within the interval calculated above.

D. We can be certain that the true proportion of individuals with BMI greater than or equal to 25 who are chronic alcohol drinkers is within the interval calculated above.

(e) The proportion of individuals living in the U.S. with Alcohol Use Disorder is 0.056. The researcher would like to know whether this is a plausible value for the proportion of individuals with BMI greater than or equal to 25 who are chronic alcohol drinkers. Based on the confidence interval calculated above, is 0.056 a plausible value?

A. Yes

B. No

5. The forensic toxicology team performs a hypothesis test to determine whether the mean BMI is greater for chronic alcohol drinkers than for not chronic alcohol drinkers. Let μ_1 and μ_2 be the mean BMI for chronic and not chronic alcohol drinkers, respectively. Their hypotheses are $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_1 > \mu_2$. Ultimately, they reject H_0 in favor of the H_a. Suppose that H_0 is in fact true. Which of the following best describes the mistake that the toxicology team has made?

   A. The Prosecutor's Fallacy

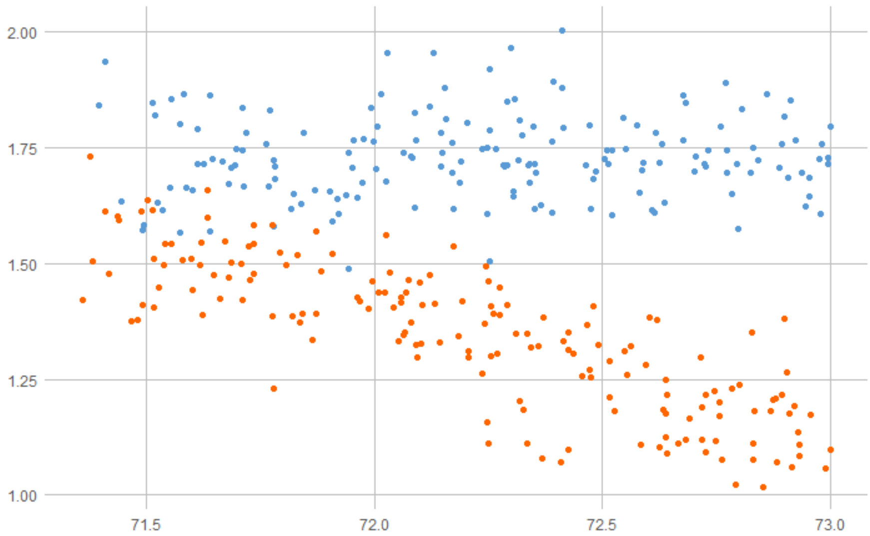
   B. A Type I Error

   C. The Defense Attorney's Fallacy

   D. A Type II Error

6. A trace analyst is interested in performing a hypothesis test to determine whether the average antimony concentrations differ between two boxes of bullets. Their goal is to detect an effect size of 0.25 with 90% power and 99% confidence. Calculate the number of samples from each box they should take to accomplish this goal.

7. A team of researchers is interested in analyzing the relationship between the elemental concentration of Silicon (Si) and the refractive index of glass. They collect 342 glass samples composed of 171 architectural glass and 171 automotive glass samples. The following scatterplot shows the refractive index vs. elemental concentration of Silicon (in ppm) for the 342 glass samples considered. The points are colored according to the type of glass (blue for architectural, orange for automotive).



(a) The researchers decide to fit a Simple Linear Regression model to the collected data. Let y_i and x_i represent the refractive index and concentration of Silicon, respectively. Which of the following defines an appropriate Simple Linear Regression model?

A. y_i is normally distributed with $E(y_i|x_i) = \beta_0 + \beta_1 x_i$ and $\text{Var}(y_i) = \sigma_i^2$.

B. y_i is normally distributed with $E(y_i|x_i) = \beta_0 + \beta_1 x_i$ and $\text{Var}(y_i) = \sigma^2$.

C. y_i is normally distributed with $E(y_i|x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^3$ and $\text{Var}(y_i) = \sigma^2$.

D. y_i is normally distributed with $E(y_i) = \beta_0 + \beta_1 x_i$ and $\text{Var}(y_i|x_i) = \sigma^2$.

(b) The following statistics are provided from the data set and fitted model. Use these to test the null hypothesis that the linear relationship between the refractive index and Silicon concentration is not significant at the $\alpha = 0.05$ level.

$$S_x^2 = 0.243$$
$$S_y^2 = 0.0551$$
$$Corr(\boldsymbol{x}, \boldsymbol{y}) = r_{x,y} = -0.357$$
$$\sum_{i=1}^{n} e_i^2 = 16.545$$

where e_i is the residual value associated with the ith observation.

i. Which of the following gives the appropriate null and alternative hypotheses?

A. $H_0 : \beta_0 = 0$ vs. $H_a : \beta_0 \neq 0$

B. $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 \neq 0$

C. $H_0 : \beta_0 \neq 0$ vs. $H_a : \beta_0 = 0$

D. $H_0 : \beta_1 \neq 0$ vs. $H_a : \beta_1 = 0$

ii. Calculate the value of the appropriate test statistic.

iii. Determine the appropriate critical value.

iv. Which of the following is the most appropriate conclusion for this hypothesis test.

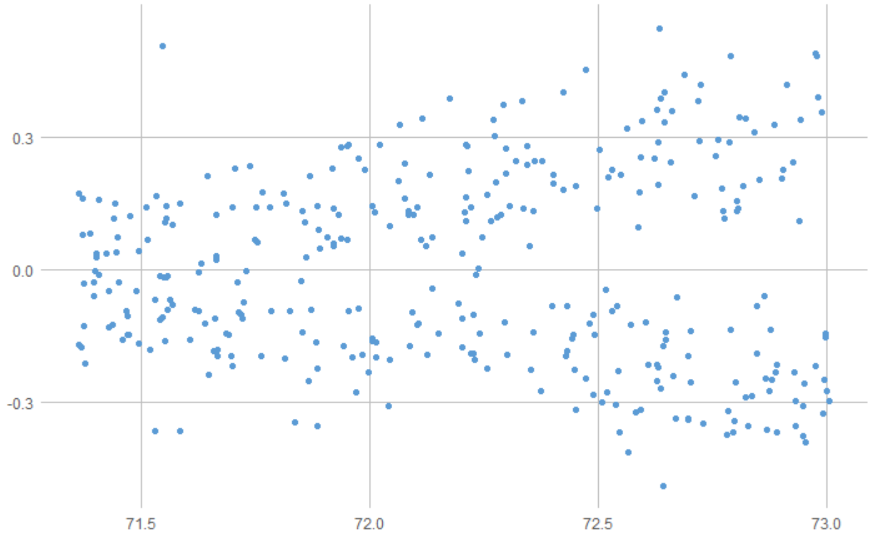
A. Reject the null meaning there is significant evidence at the $\alpha = 0.05$ level that true linear relationship between Silicon concentration and refractive index is different from 0.

B. Fail to reject the null meaning there is not significant evidence at the $\alpha = 0.05$ level that true linear relationship between Silicon concentration and refractive index is different from 0.

C. Accept the null meaning there is not significant evidence at the $\alpha = 0.05$ level that true linear relationship between Silicon concentration and refractive index is different from 0.

D. Reject the null meaning there is indisputable evidence that true linear relationship between Silicon concentration and refractive index is different from 0.

(c) A plot of the residual values on the vertical axis and Silicon concentration values on the horizontal axis is shown below.



Based on this residual plot, what do you conclude about the SLR model assumptions?

A. The independence assumption is violated.

B. The constant variance assumption is violated.

C. The 0 correlation assumption is violated.

D. The 0 expectation assumption is violated.

(d) Based on the residual plot shown above and what you know about the design of the glass sample, which of the following models would be more appropriate for the data?

A. $y_i = \beta_0 + \beta_1 x_i + \beta_2 d_i + \epsilon_i$ where $d_i = 1$ if sample i is automotive glass and 0 otherwise and $\epsilon_i \sim Normal(\mu, \sigma^2)$.

B. $y_i = \beta_0 + \beta_1 x_i + \beta_2 d_i + \beta_3 z_i + \epsilon_i$ where $d_i = 1$ if sample i is automotive glass and 0 otherwise, $z_i = x_i$ if sample i is automotive glass and 0 otherwise, and $\epsilon_i \sim Normal(0, \sigma^2)$.

C. $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ where $\epsilon_i \sim Normal(0, \sigma^2)$.

D. $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^3 + \epsilon_i$ where $\epsilon_i \sim Normal(0, \sigma^2)$.

8. A glass fragment recovered at a crime scene was compared to a glass fragment found on a suspect. Elemental glass concentration court testimony states that the probability that a randomly selected pane of glass would match the elemental concentration of a fragment found on the suspect is 0.001.

(a) Which of the following is the most appropriate interpretation of the testimony?

A. There is only about a one in 1000 chance the defendant was not at the crime scene.

B. If the glass evidence was properly collected, properly handled, and properly analyzed, there is only about a one in 1000 chance that the defendant was not at the crime scene.

C. If the glass evidence was properly collected, properly handled, and properly analyzed, there is only about a one in 1000 false match chance the glass fragment from the crime scene was not from the defendant.

D. There is a negligible chance anyone else committed crime.

(b) The examiner testifies that the glass sample from the defendant matches the glass sample taken from the crime scene. Which of the following best describes the appropriateness of the examiner's conclusion?

A. The examiner's conclusion is not appropriate because it is a statement regarding the validity of the same-source hypothesis given the evidence.

B. The examiner's conclusion is appropriate because it is a statement regarding the probative value of the evidence.

C. The examiner's conclusion is appropriate because it is very unlikely that a random match could occur.

D. The examiner's conclusion is appropriate because it is a statement regarding the validity of the same-source hypothesis given the evidence.

(c) Which of the following is the best explanation for why the Likelihood Ratio Approach might be preferred over the Two-Stage Approach?

A. The Two-Stage Approach is based entirely on the examiner's expert opinion. In contrast, the Likelihood Ratio Approach is more objective because it numerically summarizes evidential findings.

B. The Two-Stage Approach only measures the similarity between two pieces of evidence. In contrast, the Likelihood Ratio Approach actually makes a statement about the evidentiary support of the same-source hypothesis relative to the different-source hypothesis.

C. The Two-Stage Approach only compares how likely the evidence is under the same-source and different-source hypotheses. In contrast, the Likelihood Ratio Approach quantifies the how much more likely the same-source hypothesis is relative to the different-source hypothesis given the data.

D. The Two-Stage Approach only considers the probative value of evidence if the first stage finds significant similarity. In contrast, the Likelihood Ratio Approach quantifies both the similarity between two pieces of evidence and the probative value of this similarity in one number.

D.F.	α = 0.01	α = 0.05	α = 0.1
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
115	2.619	1.981	1.658
116	2.619	1.981	1.658
117	2.619	1.980	1.658
118	2.618	1.980	1.658
119	2.618	1.980	1.658
120	2.617	1.980	1.658
121	2.617	1.980	1.658
122	2.617	1.980	1.657
123	2.616	1.979	1.657
124	2.616	1.979	1.657
125	2.616	1.979	1.657
335	2.591	1.967	1.649
336	2.591	1.967	1.649
337	2.590	1.967	1.649
338	2.590	1.967	1.649
339	2.590	1.967	1.649
340	2.590	1.967	1.649
341	2.590	1.967	1.649
342	2.590	1.967	1.649
343	2.590	1.967	1.649
344	2.590	1.967	1.649
345	2.590	1.967	1.649
Std. Normal	2.576	1.960	1.645

Figure 1: t and standard normal critical values associated with different α levels (i.e., $(1-\alpha/2)$ quantiles)